

Multicriteria Fractional Model for Feed Formulation: Economic, Nutritional and Environmental Criteria

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Abstract

The traditionally used linear programming model for feed formulation has sought the least-cost combination of ingredients that satisfies a specific level of nutritional requirements. Together with the search for the lowest possible cost, other aspects such as maximising diet efficiency in relation to proposed production objectives and minimising any excess that may lead to unacceptable damage to the environment, are other factors that are gaining in importance in the world of animal nutrition. Taking into account all these factors forces us not only to bear in mind the individual concentrations of each of the nutrients in the feed but also the ratios of these with other nutrients.

In this work we show how a multi-objective fractional programming model is better adapted to current needs in feed design than the traditionally used least cost linear model.

We also show how the model can be solved using the interactive multi-goal programming method with existing linear optimisation software.

Keywords: Multi-objective fractional programming; Diet formulation; Pig-nutrition; Environment.

1. Introduction

Pig breeding is a highly competitive industry, characterised by low profit margins. Within this production context, economic efficiency is the key element to ensuring long term profitability. In production systems, feeding accounts for the greatest single item in total cost production and minor reductions in costs linked to fattening may have significant impact on production profitability. It is thus crucial to clarify the nutritional requirements of the animals because incorrect or inadequate nutrition may lead to a reduction in pig growth, whereas any excess in nutrients causes waste that can prove costly. In the case of protein and certain other minerals such as phosphorus,

this waste may be a major source of environmental damage [1-3].

In the present work, we build on previous approaches reported in the literature to deal with the formulation problem in a more complete way [4-6]. We consider economic and environmental objectives, incorporate advanced nutritional concepts in ratio form, and use an interactive method. We propose the design of a feed for pigs using multiple objective fractional programming encompassing the cost of the feed, the lysine/energy ratio, deviation with regard to the ideal values of the percentage content of amino acids in the protein, and the amount of phosphorus. Having formulated the model, we explain how it can be solved using Interactive Multiple Goal Programming (IMGP) method [7]. To illustrate its use we perform an actual application to the formulation of a feed for growing pigs. We discuss the results and conclude with a brief discussion on the adequacy of our model.

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2. Formulation of the model

Our aim is to propose a qualitative and quantitative composition of animal diet to provide the best results with regard to production goals (increase in weight, lean meat deposit, etc.) at the lowest possible cost, with as little environmental impact as possible [8-10].

Let $x = (x_1, x_2, \dots, x_n)$ where $x_j, j = 1, \dots, n$ denotes the proportion of ingredient j in the diet and n is the total number of ingredients available.

The region of variation of x will be bounded by two types of constraints:

- Constraints that ensure chemical composition of the diet adapted to the needs of the animal. These define the maximum and minimum amounts of nutrient content such as calcium, protein, fibre, available phosphorus, dry matter, energy, etc.

$$\underline{b}_i \leq \sum_{j=1}^n a_{ij}x_j \leq \overline{b}_i, \quad i = 1, \dots, k$$

k being the number of nutrients considered, a_{ij} the amount of nutrient i in ingredient j , \underline{b}_i and \overline{b}_i the lower and upper bounds, respectively, of nutrient i in the diet.,

- Constraints that restrict the proportion of certain ingredients in the diet,

$$x_j \leq s_j, j = 1, \dots, n$$

where s_j is the maximum proportion of ingredient j in the diet.

To evaluate a diet, four criteria are considered:

1. *Economic criterion.* The cost of the feed:

$$f_1 = \sum_{j=1}^n c_j x_j,$$

where s_j being the unit price of ingredient j .

2. *Nutritional criterion.* The lysine/energy ratio:

$$f_2 = \frac{\sum_{j=1}^n l_j x_j}{\sum_{j=1}^n e_j x_j},$$

where l_j is the amount of lysine in ingredient j and e_j is the amount of energy in ingredient j .

3. *Nutritional-ecological criterion.* Maximum deviation, with regard to values considered ideal, of the percentage content of the amino acids

methionine + cystine, threonine and tryptophan in comparison with lysine content

$$f_3 = \max \left(\left| \frac{\sum_{j=1}^n mc_j x_j}{\sum_{j=1}^n l_j x_j} - 0,5 \right|, \left| \frac{\sum_{j=1}^n t_j x_j}{\sum_{j=1}^n l_j x_j} - 0,66 \right|, \left| \frac{\sum_{j=1}^n tp_j x_j}{\sum_{j=1}^n l_j x_j} - 0,18 \right| \right),$$

mc_j being the amount of methionine + cystine in ingredient j , t_j the amount of threonine in ingredient j , tp_j the amount of tryptophan in ingredient j , and 50, 66, 18 are the ideal percentages of methionine + cystine, threonine and tryptophan, respectively, with regard to lysine [1].

4. *Ecological criterion.* Having ensured in the constraints that the amount of phosphorus available in the diet meets the needs of the animal, the fourth factor to be taken into account is the total amount of phosphorus in the feed:

$$f_4 = \sum_{j=1}^n p_j x_j,$$

p_j being the amount of phosphorus in ingredient j .

The formulation of the fractional multiobjective problem (FMP) is the following:

$$\text{Min } f_1, \text{Min } f_2, \text{Min } f_3, \text{Min } f_4 \text{ (FMP)}$$

s.t. $x \in S$ where

$$S = \left\{ x = (x_1, \dots, x_n) \in \mathbb{R}^n / \underline{b}_i \leq \sum_{j=1}^n a_{ij}x_j \leq \overline{b}_i, i = \overline{1, k}; x_j \leq s_j, j = \overline{1, n} \right\}$$

3. Resolution of the model

In the model proposed above, criteria f_2 and f_3 are non-linear, fractional and minimax fractional, respectively, making their operative treatment extremely difficult. The efficacy of some of the methods that pursue the determination of all or part of the set of efficient solutions is limited because of the special character of these criteria [11, 12].

The decision method chosen to resolve this model more easily is the Interactive Multiple Goal Programming (IMGP) method proposed by [7].

In broad terms, IMGP works as follows: The first step is to optimise successively each of the four objectives separately over S . Next, the first potency matrix, P_1 , is built

$$P_1 = \begin{pmatrix} f_1^* & f_2^* & f_3^* & f_4^* \\ f_1 & f_2 & f_3 & f_4 \end{pmatrix}$$

This matrix summarizes the criteria values in the four solutions obtained. The first row contains the best or ideal criteria values, f_i^* , $i = 1, \dots, 4$, whereas the second row shows the worst or pessimistic criteria values, \bar{f}_i , $i = 1, \dots, 4$. Each column of the potency matrix represents the range within which it is possible to define the goals for each criterion in the next iteration.

In the following step, the decision maker must indicate which of the values they prefer to improve first. A new potency matrix, P_2 , is then built. To obtain this matrix each of the objectives is individually optimised, subject to the set of constraints of the initial problem, S , and the additional constraint that f_p reach the new value. The first row of P_2 contains the best or ideal criteria values in this iteration, whereas the second row shows the worst.

Once P_2 is determined, the decision maker must evaluate if the improvement in f_p makes up for the

changes in the potentially attainable values, f_q^* , $q \neq p$ of the remaining criteria. If this is not the case, the decision maker may review his or her wishes with regard to the change in value of f_p . If it is, the new value of f_p is accepted and the decision maker may go on improving another or even the same criterion. The column of P_2 associate with that criterion represents the range within which it is possible to define the target for the same in the next iteration. In each iteration either a new constraint is added or a constraint that is already being used is updated. Thus, the set of feasible solutions is reduced in each iteration. The iterative process continues until the two rows of the potency matrix are equal. In this case, the solution obtained is efficient, i.e., there is not another feasible solution that ameliorates the achievement of one criterion without worsening the achievement of at least one other criterion.

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Table 1. Nutrient content of feed ingredients

Ingredients	CF (%)	MC (%)	Tp (%)	T (%)	Ca (%)	P (%)	AP (%)	DM (%)	CP (%)	L (%)	DE (MJ/kg)
x1 Barley	4.50	0.43	0.13	0.37	0.06	0.36	0.13	90.20	11.30	0.40	13.25
x2 Wheat	2.80	0.46	0.13	0.34	0.04	0.35	0.18	89.40	11.60	0.33	14.34
x3 Corn	2.50	0.33	0.06	0.27	0.02	0.27	0.05	86.30	7.70	0.22	14.42
x4 Alfalfa	24.70	0.45	0.31	0.70	1.75	0.30	0.26	91.20	16.70	0.73	7.82
x5 Cassava meal	6.10	0.06	0.02	0.07	0.24	0.10	0.03	88.80	2.50	0.09	13.29
x6 Soybean meal 44	5.60	1.28	0.59	1.75	0.29	0.61	0.19	88.00	44.00	2.88	13.79
x7 Fish meal	1.00	2.36	0.65	2.65	4.50	2.77	2.45	92.00	62.40	4.75	15.26
x8 Gluten feed	8.00	0.83	0.13	0.74	0.16	0.80	0.22	88.60	19.00	0.62	11.37
x9 Calcium carbonate	-	-	-	-	38.30	0.02	-	98.00	-	-	-
x10 Lysine 78%	-	-	-	-	0.04	-	-	98.50	95.00	78.00	20.56
x11 Sunflower meal 30	22.50	1.25	0.43	1.06	0.35	1.00	0.17	89.30	30.50	1.06	9.20
x12 Animal fat	-	-	-	-	-	-	-	-	-	-	34.07
x13 Beet pulp	17.80	0.22	0.10	0.47	0.98	0.11	0.04	89.70	10.10	0.59	11.29
x14 Lupin	14.50	0.66	0.25	0.99	0.23	0.32	0.13	90.80	30.70	1.46	13.58
x15 Peas	5.70	0.56	0.19	0.82	0.10	0.40	0.15	86.70	21.50	1.55	13.79
x16 Rye	2.30	0.37	0.10	0.30	0.05	0.34	0.15	89.40	8.90	0.34	13.79
x17 Oats	10.50	0.47	0.14	0.35	0.07	0.33	0.09	89.60	10.50	0.40	11.33
x18 Rapeseeds meal	12.90	1.82	0.45	1.70	0.70	1.20	0.30	91.00	37.90	2.16	12.25
x19 Triticale	2.70	0.49	0.13	0.36	0.05	0.34	0.17	89.00	12.00	0.42	13.96
x20 Dicalcium phosphate	-	-	-	-	32.00	18.00	-	99.40	-	-	-

CF: crude fibre; MC: methionine + cystine; Tp: tryptophan; T: threonine; Ca: calcium; P: phosphorus; AP: available phosphorus; DM: dry matter; CP: crude protein; L: lysine; DE: digestible energy. Source: Normas FEDNA (1999) [13].

Table 2. Limits on ingredients content and unitary costs

Ingredients	Bounds (%)	Cost (€MT)
Barley	–	122.01
Wheat	40.00	126.21
Corn	40.00	134.63
Alfalfa	5.00	132.22
Cassava meal	22.00	151.45
Soybean meal 44	–	172.49
Fish meal	4.00	408.68
Gluten feed	8.00	122.61
Calcium carbonate	–	300.51
Lysine 78%	0.65	2434.04
Sunflower meal 30	6.00	118.40
Animal fat	4.00	384.69
Beet pulp	5.00	152.66
Lupin	10.00	150.25
Peas	15.00	159.27
Rye	20.00	136.73
Oats	12.00	142.14
Rapeseeds meal	8.00	135.23
Triticale	30.00	132.22
Dicalcium phosphate	–	300.51

Source: Normas FEDNA (1999) [13].

Table 3. Nutritional requirement

Nutrients	CF (%)	Ca (%)	AP (%)	DM (%)	CP (%)	DE
Lower bound	–	0.50	0.19	87.00	15.50	13.00
Upper bound	4.00	–	–	95.00	20.00	–

Source: NRC (1998) [14].

4. Application of the model and results

To illustrate the model described, consider a real-life problem of searching for the most suitable diet for a growing pig of 50–80 kg. Twenty ingredients and eleven nutrients have been used in the study. The nutrient contents of the ingredients were derived from the feed composition tables [13] and are presented in Table 1. Table 2 contains limits on ingredients in the feed and the unitary cost of every ingredient. Finally, Table 3 contains nutrient requirements for pigs recommended by the NRC [14].

The problem has been solved using the LINDO computer program and Excel 7.0 spreadsheets. An additional constraint was introduced, requiring the sum of all the ingredients to be equal to unity, by which the solution appears as percentage.

A member of the Department of Animal Production at the University of Cordoba, who assessed the solutions obtained in each iteration, acted as the decision maker.

The interactive process is described below

•Iteration 1

Each of the criteria was first optimised individually over S, without any restrictions on the values of the others criteria. Table 4 gives the starting iteration of the model. The first row shows the values of the criteria in the diet for which the cost criteria is minimised. The next rows of this table refer to the three other diets for which the other criteria were optimised.

Table 4. The optimal solutions of single criterion problems

Problem	Criterion	Cost	L/E	Max. dev.	P
Min.	Cost	135.648	0.51	0.4243	0.4656
Max.	L/E	180.003	1.27	0.2211	1.1130
Min.	Max. dev.	170.386	0.88	0.0161	1.1390
Min.	P	159.328	0.81	0.2378	0.3607

The first potency matrix P_1 summarizes the criteria values after the first iteration,

$$P_1 = \begin{pmatrix} 135.648 & 1.27 & 0.0161 & 0.3607 \\ 180.003 & 0.51 & 0.4243 & 1.1390 \end{pmatrix}$$

The first row contains the best values of the criteria whereas the second row shows the worst values.

The four solutions found were thought as unsatisfactory by the decision maker, as was expected.

•Iteration 2

The decision maker first considered that the lysine/energy ratio level should not be less than 0.69 to optimise the growth potential of the animal. For this reason we started the second iteration in which we imposed an additional restriction $f_2 \geq 0.69$. After the optimising process, the impact of this additional restriction was summarized in a new potency matrix, P_2 ,

$$P_2 = \begin{pmatrix} 140.396 & 1.27 & 0.0161 & 0.3607 \\ 180.003 & 0.69 & 0.2211 & 1.139 \end{pmatrix}$$

The decision maker felt that the minimum value set for the lysine/energy ratio made up for the drop in possible cost improvement.

•Iteration 3

The decision maker considered that the cost of the diet should never be greater than 150.25. A new constraint, $f_1 \geq 150.25$, was added to the set of constraints of the previous iteration, and P_3 was calculated:

$$P_3 = \begin{pmatrix} 140.396 & 1.01 & 0.0297 & 0.3669 \\ 150.25 & 0.69 & 0.2156 & 0.4624 \end{pmatrix}$$

The decision maker considered that the maximum value set for the cost makes up for the reduction in

the possible improvements of the remaining objectives.

•Iteration 4

The decision maker felt that the maximum deviation in the percentage composition of the amino acids with regard to the ideal values should not be greater than 0.03, to maintain the required balance in the animal and control the nitrogen pollution.

A new constraint, $f_3 \leq 0.03$, was added to the set of constraints in the previous iteration and a new potency matrix, P_4 , was calculated:

The decision maker did not consider the shifts in the potency matrix justified. Comparing P_3 with P_4 , the decision maker judged that the best values for the cost and the phosphorous were too high in P_4 . He decided to set a new but less ambitious value for the third criterion. The new value for this criterion was 0.06. The constraint $f_3 \leq 0.03$ was substituted by $f_3 \leq 0.06$.

Therefore, the revised fourth potency matrix was

$$P_4 = \begin{pmatrix} 143.461 & 0.938 & 0.0297 & 0.375 \\ 150.25 & 0.69 & 0.06 & 0.48 \end{pmatrix}$$

The decision maker accepted the trade-offs between the criteria.

•Iteration 5

The decision maker considered that the cost should not be greater than 146.166 in order for the diet to be more attractive from an economic perspective. A new constraint, $f_1 \leq 146.166$, was added to the set of constraints in the previous iteration, and P_5 was calculated:

$$P_5 = \begin{pmatrix} 143.461 & 0.83 & 0.0428 & 0.3824 \\ 146.166 & 0.69 & 0.06 & 0.4493 \end{pmatrix}$$

The decision maker considered that the maximum value set for the cost makes up for the reduction in the possible improvements of the remaining objectives.

•Iteration 6

The decision maker considered that total phosphorus in the diet should not exceed 0.39. A new constraint was added, $f_4 \leq 0.39$, and a new potency matrix was calculated:

$$P_6 = \begin{pmatrix} 144.408 & 0.72 & 0.045 & 0.3824 \\ 146.166 & 0.69 & 0.06 & 0.39 \end{pmatrix}$$

Once again, the decision maker considered the changes in the potency matrix to be acceptable.

•Iteration 7

Finally, the decision maker considered the worst value of the cost, lysine/energy ratio as well as the phosphorus amount in the anterior solutions to be acceptable and chose to reduce the maximum

deviation to the minimum possible value. A new constraint $f_3 \leq 0.045$, was added and P_7 was calculated:

$$P_7 = \begin{pmatrix} 146.166 & 0.69 & 0.045 & 0.39 \\ 146.166 & 0.69 & 0.045 & 0.39 \end{pmatrix}$$

The iterative process ends because the two rows of the potency matrix are equal. The rows of the last potency matrix reflect the aspiration levels for each of the objectives in the final solution that have been interactively obtained from the decision maker. This solution is efficient, i.e., no other solution can be found that ameliorates the achievement of one criterion without worsening the achievement of at least one other criterion.

The decision maker's preferred diet composition is shown in Table 5.

Table 6 shows the nutritional content of the diet.

Table 5. Values of ingredients in the decision maker's preferred diet

Ingredients	%
Cassava meal	17.53
Soybean meal 44	15.48
Fish meal	0.03
Calcium carbonate	3.33
Lysine 78%	0.05
Animal fat	0.56
Peas	15.00
Rye	20.00
Triticale	27.71
Dicalcium phosphate	0.31

Table 6. Nutrient content in the decision maker's preferred diet

CF (%)	MC (%)	Tp (%)	T (%)	Ca (%)	P (%)	AP (%)	DM (%)	CP (%)	L (%)	DE (MJ/kg)
4.00	0.50	0.184	0.57	1.50	0.39	0.19	88.38	15.65	0.92	13.37

4. Conclusions

While assessing the adequacy of our model could take up another paper, a few brief comments could be made here. A mathematical model is only a way of approaching a problem. For solving a problem, we construct a model of the real world, solve the problem in the model, and then implement the solution obtained in the real world. How good the model is can be judged by several

criteria. The following comments try to relate our model to some of the most important criteria. The first criterion could be the approximation of the model to the real world, i.e., how many aspects of the reality involved are captured by the model. Despite its wide-spread use, in practice linear programming has significant restrictions as regards its adaptability to real nutritional and environmental problems in feed design. In this work, we show how a non-linear multicriteria

model, with cost minimising criteria, maximising lysine/energy ratio, minimising maximum deviation with regard to ideal composition of protein, and minimising the amount of phosphorus, provides a more suitable framework for solving the kind of problem that nutritionist perceive and manage today.

A second criterion to judge the model could be how operational and user-friendly it is. The problem is solved using the interactive multiple goal programming (IMGP) method. In each IMGP iteration, each of the objectives is optimised separately, the others acting as constraints allowing us to offset the technical difficulties arising out of the presence of fractional and minimax objectives, thus enabling the problem to be dealt with more easily from the computational viewpoint. Another advantage of IMGP in comparison with other methods is that the decision maker need not specify a priori the aspiration levels for each objective as these are obtained during the interactive process. A minimum piece of information is required from the ration formulator.

A third criterion could be the quality of the solution obtained. In this sense, single objective mathematical programming model are a special kind of models because they try, for a full representation of a reality, to identify the optimum solution. Contrary to this, MCMP models acknowledge that reality is more complex and there is no such kind of optima. The quality of the solutions, in this context, can be approached by new information provided about the real world and the implications of implementing them. Our model shows that it is not possible to reduce nitrogen and phosphorus excretions without increasing cost. However, nitrogen and phosphorus excretion can be significantly reduced by just a small increase in cost. This type of information about the trade-offs between criteria is a valuable product of our approach.

The fourth criterion is generality. Has the model the capacity to be extended to tackle other situations of the real world? In general, MCMP models are flexible frameworks in which generality is an intrinsic property, in contrast to ad-hoc models. Although our work has focused on pig production, the model may easily be adapted to other animal species. And in particular, advanced nutrition theory is full of concepts that

can be treated as ratios, making the fractional framework a natural way of dealing with them.

Fifth, and in our opinion, one of the most important criteria in this case, is the level of understanding the user has or can acquire, of the principles involved in the model. It is obvious that while the sophistication of the mathematical apparatus of the model increases, this level of understanding may decrease. This is an important aspect of the confidence that a user can have in the model. The users of our model are ration formulators, usually expert and scientific nutritionists. At the beginning, for users adapted to an easily understandable method such as linear programming, to change their mathematical paradigm can have a high opportunity cost.

But the increasing sophistication of nutritional concepts and theories will push ration formulators, sooner or later, to accept such changes. The application of multicriteria methods in animal nutrition will be accelerated if researchers from both fields of nutrition and mathematical programming engage in the other field, or if interdisciplinary teams become involved in these issues.

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